
ANALYSIS OF INVENTORY MODEL FOR DECAYING ITEMS WITH INFLATION BELOW PERMISSIBLE DELAY IN PAYMENTS

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ABSTARCT

In this paper we will develop an EOQ model for deteriorating items with quadratic demand and permissible delay in payments. Sensitivity with respect to parameters has been carried out. The demand rate is such that as the inventory level increases, it helps to increase the demand for the inventory under consideration. While as the time passes, demand is depends upon the various factors. The results show that with the increase/ decrease in the parameter values there is corresponding increase/ decrease in the value of cost.

KEY WORDS: Inventory, deteriorating

INTRODUCTION & REVIEW OF LITERATURE

Deterioration of goods is a common phenomenon in daily life. Therefore controlling and maintaining the inventory of deteriorating items becomes an important factor for decision makers. Whitin [31] first studied the inventory model for fashion goods deteriorating at the end of prescribed period. Ghare and Schrader [11] developed an EOQ model with constant rate of deterioration. Covert and Philip [9] extended this model by considering variable rate of deterioration. The model was further extended by Shah [26] by considering shortages. The related works are found in (Nahmias [23], Raffat [24], Goyal and Giri [13], Wu et al. [32]).

In all the inventory models for deteriorating items it is assumed that deterioration starts as soon as the retailer receives the inventory. But most of the goods have a span of maintaining quality or the original condition in real situation. During that period, there was no occurrence of deterioration. This phenomenon is termed as “non-instantaneous deterioration”. A non-instantaneous deteriorating items inventory models with price discount was developed by Jeyaraman and Sugapriya [19]. Many times customers would like to wait for backlogging during the shortage period but the others would not. Therefore, the opportunity cost due to lost sales is to be taken into consideration in the modeling. Chang and Dye [5] considered an EOQ model for deteriorating items with time varying demand and partial backlogging. They were the first to give a definition for time dependent partial backlogging rate. Sana [25] considered lot size inventory model with time varying determination and partial backlogging. Begum et al. [2] considered an inventory model for deteriorating items with quadratic demand and partial backlogging.

Goyal [12] first considered the economic order quantity model under the condition of permissible delay in payments. Goyal's [12] model was extended by Aggarwal and Jaggi [1] for deteriorating items. Jamal et al. [17] further extended Aggarwal and Jaggi's [1] model to consider shortages. The related works are found in (Chung and Dye [7], Jamal et al. [18], Chung et al. [8], Chang et al. [4]). The first economic order quantity model by considering the effect of inflation was developed by Buzacott [3]. Su et al. [29] developed model under inflation for stock dependent consumption rate and exponential decay. The EOQ model for ameliorating / deteriorating items with time varying demand pattern over a finite planning horizon taking into account the effect of inflation and time value of money was considered by Moon et al. [22]. Mishra et al. [21] developed the model for deterministic perishable items with variable type demand rate under infinite time horizon and constant deterioration. Datta and Pal [10] considered the economic order quantity model incorporating the effects of time value of money and shortages. Demand was considered as linear function of time. Hariga [14] extended Datta and Pal's [10] model by relaxing the assumption of equal inventory carrying time during each replenishment cycle and modified their

mathematical formulation. Hariga and Ben-Daya [15] extended Hariga's [14] model by removing the restriction of equal replenishment cycle and provided two solution procedures with and without shortages. The related works are found in (Hou [16], Chern et al. [6], Yang et al. [30], Singh et al. [28], Liao et al. [20], Singh [27]). Raman Patel [33] An EOQ model for deteriorating items with quadratic demand under inflation and permissible delay in payments is considered. Holding cost is linear function of time. Shortages are allowed and are partially backlogged. Numerical example is taken and sensitivity is also carried out to support the model.

ASSUMPTIONS AND NOTATIONS

The mathematical models of the two warehouse inventory problems are based on the following assumptions and notations:

Assumptions:

- The inventory system involves a single type of items.
- Demand rate is dependent on time and stock level.
- Deterioration rate is taken as Kt .
- Shortages are not permitted.
- The replenishment rate is instantaneous.
- Lead time is neglected.
- Permissible delay in payment to the supplier by the retailer is considered. The supplier offers different discount rates of price at different delay periods.
- Planning horizon is infinite.
- Inflation and time value of money is considered.

Notations:

- $D = a + bt + cI(t)$ Time and Stock dependent demand
- C_0 = Ordering cost
- C_h = holding cost per unit time, excluding interest charges
- C_p = purchasing cost which depends on the delay period and supplier's offers
- p = selling price per unit
- M = permissible delay period
- $M_i = i^{th}$ permissible delay period in settling the amount
- i = discount rate (in %) of purchasing cost at i -th permissible delay period.
- i_e = rate of interest which can be gained due to credit balance
- i_c = rate of interest charged for financing inventory
- T = length of replenishment
- $AP_1(T, M_i) =$ average profit of the system for $T \geq M_i$
- $AP_2(T, M_i) =$ average profit of the system for $T \leq M_i$
- Q_0 = Initial lot size

MODEL FORMULATION AND SOLUTION:

The cycle starts with initial lot size Q_0 and ends with zero inventory at time $t=T$. Then the differential equation governing the transition of the system is given by

$$\frac{dI(t)}{dt} = -Kt - (a + bt + cI(t)), \quad 0 \leq t \leq T \quad \dots (1)$$

With boundary condition $I(0) = Q_0$

The purchasing cost at different delay periods are

$$C_p = \begin{cases} C_r(1 - \delta_1), M = M_1 \\ C_r(1 - \delta_2), M = M_2 \\ C_r(1 - \delta_3), M = M_3 \\ \infty, M > M_3 \end{cases}$$

Where C_r = maximum retail price per unit.

And M_i ($i=1,2,3$) = decision point in settling the account to the supplier at which supplier offers δ % discount to the retailer.

Now two cases may occur:

1. When $T \geq M$
2. When $T < M$

Case 1: when $T \geq M$

Solving the equation (1), we get

$$\frac{dI(t)}{dt} + KtI(t) = -(a + bt + cI(t))$$

Using the boundary condition $I(0) = Q_0$, we get

$$c = Q_0$$

Therefore the solution of equation (1) is

$$I(t) = \left\{ Q_0 - at - \frac{(a+b)}{2}t^2 - \left(\frac{aK}{2} + b \right) \frac{t^3}{3} - \frac{bK}{8}t^4 \right\} e^{-t-Kt^2/2} \quad 0 \leq t \leq T \quad \dots (2)$$

In this case it is assumed that that the replenishment cycle T is larger than the credit period M .

The holding cost, excluding interest charges is

$$\begin{aligned}
 HC &= C_h \int_0^T I(t) e^{-rt} dt \\
 HC &= C_h \left[\left\{ Q_0 T - \frac{a}{2} T^2 - \frac{(a+b)}{6} T^3 - \left(\frac{aK}{2} + b \right) \frac{T^4}{12} - \frac{bK}{40} T^5 \right\} \right. \\
 &\quad \left. - (1+r) \left\{ \frac{Q_0}{2} T^2 - \frac{a}{3} T^3 - \frac{(a+b)}{8} T^4 - \left(\frac{aK}{2} + b \right) \frac{T^5}{15} - \frac{bK}{48} T^6 \right\} \right. \\
 &\quad \left. - \frac{K}{2} \left\{ \frac{Q_0}{3} T^3 - \frac{a}{4} T^4 - \frac{(a+b)}{10} T^5 - \left(\frac{aK}{2} + b \right) \frac{T^6}{18} - \frac{bK}{56} T^7 \right\} \right] \quad \dots (3)
 \end{aligned}$$

The cost of financing inventory during time span $[M, T]$ is

$$\begin{aligned}
 FC &= i_c C_p \int_M^T I(t) e^{-r(M+t)} dt \\
 FC &= i_c C_p \left[(1-rM) \left\{ Q_0 T - \frac{a}{2} T^2 - \frac{(a+b)}{6} T^3 - \left(\frac{aK}{2} + b \right) \frac{T^4}{12} - \frac{bK}{40} T^5 \right\} \right. \\
 &\quad \left. - (1+r) \left\{ \frac{Q_0}{2} T^2 - \frac{a}{3} T^3 - \frac{(a+b)}{8} T^4 - \left(\frac{aK}{2} + b \right) \frac{T^5}{15} - \frac{bK}{48} T^6 \right\} \right. \\
 &\quad \left. - \frac{K}{2} \left\{ \frac{Q_0}{3} T^3 - \frac{a}{4} T^4 - \frac{(a+b)}{10} T^5 - \left(\frac{aK}{2} + b \right) \frac{T^6}{18} - \frac{bK}{56} T^7 \right\} \right. \\
 &\quad \left. - (1-rM) \left\{ Q_0 M - \frac{a}{2} M^2 - \frac{(a+b)}{6} M^3 - \left(\frac{aK}{2} + b \right) \frac{M^4}{12} - \frac{bK}{40} M^5 \right\} \right. \\
 &\quad \left. + (1+r) \left\{ \frac{Q_0}{2} M^2 - \frac{a}{3} M^3 - \frac{(a+b)}{8} M^4 - \left(\frac{aK}{2} + b \right) \frac{M^5}{15} - \frac{bK}{48} M^6 \right\} \right. \\
 &\quad \left. + \frac{K}{2} \left\{ \frac{Q_0}{3} M^3 - \frac{a}{4} M^4 - \frac{(a+b)}{10} M^5 - \left(\frac{aK}{2} + b \right) \frac{M^6}{18} - \frac{bK}{56} M^7 \right\} \right] \quad \dots (4)
 \end{aligned}$$

Opportunity gain due to credit balance during time span $[0, M]$ is

$$Opp.Gain = i_e p \int_0^M (M-t)(a+bt+cI(t))e^{-rt} dt$$

$$Opp.Cost = i_e p \left[(a+bM) \frac{e^{-rM}}{r^2} - 2b \frac{e^{-rM}}{r^3} + \frac{aM}{r} - \frac{(a-bM)}{r^2} + \frac{2b}{r^3} \right] \dots (5)$$

Therefore, the total cost is given by

TC_{1i}=Purchasing Cost +holding cost +ordering cost +interest charged-interest earned for M∈{M₁,M₂,M₃}

$$TAC_{1i} = \frac{1}{T} TC_{1i} \dots (6)$$

Case 2 when T < M

In this case, credit period is larger than the replenishment cycle consequently cost of financing inventory is zero. The holding cost, excluding interest charges is

$$HC = C_h \int_0^T I(t)e^{-rt} dt$$

$$HC = C_h \left[\left\{ Q_0 T - \left(\frac{a}{2} T^2 + \frac{b}{6} T^3 + \frac{aK}{24} T^4 + \frac{bK}{40} T^5 \right) \right\} \right. \\ \left. - r \left\{ \frac{Q_0}{2} T^2 - \left(\frac{a}{3} T^3 + \frac{b}{8} T^4 + \frac{aK}{30} T^5 + \frac{bK}{48} T^6 \right) \right\} \right. \\ \left. - \frac{K}{2} \left\{ \frac{Q_0}{3} T^3 - \left(\frac{a}{4} T^4 + \frac{b}{10} T^5 + \frac{aK}{36} T^6 + \frac{bK}{56} T^7 \right) \right\} \right] \dots (7)$$

Opportunity gain due to credit balance during time span [0,M] is

$$Opp.Gain = i_e p \left[\int_0^T (T-t)(a+bt)e^{-rt} dt + \int_0^T (M-T)(a+bt)e^{-rt} dt \right]$$

$$= i_e p \left[\int_0^T \left\{ aT + (bT-a)t - bt^2 \right\} e^{-rt} dt + (M-T) \int_0^T \left\{ a+bt \right\} e^{-rt} dt \right] \dots (8)$$

Therefore the total cost during the time interval T is given by

TC_{2i}=Purchasing cost +holding cost +ordering cost-interest earned (Opp. cost)

$$TAC_{2i} = \frac{1}{T} TC_{2i} \dots (9)$$

Now, our aim is to determine the optimal value of T and M such that TAC(T,M) is minimized where

$$TAC(T, M) = Inf. \begin{cases} TAC_{1i}(T, M), TAC_{2i}(T, M) \\ \text{where, } M \in (M_1, M_2, M_3) \end{cases} \dots (10)$$

Special case:

Case 1 when there is no deterioration, i.e. K=0, then

$$I(t) = \left\{ Q_0 - \left(at + \frac{b}{2} t^2 \right) \right\}, \quad 0 \leq t \leq T$$

$$FC = i_c C_p \left[\left\{ Q_0 T - \left(\frac{a}{2} T^2 + \frac{b}{6} T^3 \right) \right\} - rM \left\{ Q_0 - \left(aT + \frac{b}{2} T^2 \right) \right\} \right. \\ \left. - \frac{K}{2} \left\{ \frac{Q_0}{3} T^3 - \left(\frac{a}{4} T^4 + \frac{b}{10} T^5 \right) \right\} - r \left\{ \frac{Q_0}{2} T^2 - \left(\frac{a}{3} T^3 + \frac{b}{8} T^4 \right) \right\} \right. \\ \left. - \left\{ Q_0 M - \left(\frac{a}{2} M^2 + \frac{b}{6} M^3 \right) \right\} + rM \left\{ Q_0 - \left(aM + \frac{b}{2} M^2 \right) \right\} \right]$$

$$+ \frac{K}{2} \left\{ \frac{Q_0}{3} M^3 - \left(\frac{a}{4} M^4 + \frac{b}{10} M^5 \right) \right\} + r \left\{ \frac{Q_0}{2} M^2 - \left(\frac{a}{3} M^3 + \frac{b}{8} M^4 \right) \right\} \Bigg] \\
 Opp.Cost = i_e p \left[(a + bM) \frac{e^{-rM}}{r^2} - 2b \frac{e^{-rM}}{r^3} + \frac{aM}{r} - \frac{(a - bM)}{r^2} + \frac{2b}{r^3} \right]$$

Case 2: when the demand rate is constant means b=0

$$I(t) = \left\{ Q_0 - \left(at + \frac{aK}{6} t^3 \right) \right\} e^{-Kt^2/2} \quad 0 \leq t \leq T$$

$$HC = C_h \left[\left\{ Q_0 T - \left(\frac{a}{2} T^2 + \frac{aK}{24} T^4 \right) \right\} - r \left\{ \frac{Q_0}{2} T^2 - \left(\frac{a}{3} T^3 + \frac{aK}{30} T^5 \right) \right\} - \frac{K}{2} \left\{ \frac{Q_0}{3} T^3 - \left(\frac{a}{4} T^4 + \frac{aK}{36} T^6 \right) \right\} \right]$$

$$Opp.Cost = i_e p \left[a \frac{e^{-rM}}{r^2} + \frac{aM}{r} - \frac{a}{r^2} \right]$$

Table 1: Variation in TC with the variation in a

| a | T | TC(10⁵) |
|----------|----------|---------------------------|
| 70 | 35.7643 | 6.5247 |
| 80 | 34.1886 | 6.1436 |
| 90 | 33.9981 | 5.7245 |
| 100 | 32.0002 | 5.2681 |
| 110 | 32.8266 | 5.8957 |
| 120 | 30.1724 | 5.3541 |
| 130 | 25.9610 | 4.5232 |

Table 2: Variation in TC with the variation in b

| b | T | TAC(10⁵) |
|----------|----------|----------------------------|
| 30 | 25.235 | 10.2954 |
| 35 | 28.1254 | 10.1118 |
| 40 | 30.4457 | 9.9725 |
| 45 | 33.1896 | 8.1829 |
| 50 | 33.7832 | 7.2681 |
| 55 | 34.1457 | 7.0075 |
| 60 | 34.8485 | 6.3221 |
| 65 | 35.9517 | 6.1882 |
| 70 | 36.1725 | 6.0914 |
| 75 | 38.8954 | 5.3236 |

Table 3: Variation in TC with the variation in C_h

| C_h | T | TAC(10^5) |
|-------|---------|---------------|
| 0.020 | 18.7241 | 1.8954 |
| 0.025 | 21.9154 | 2.3112 |
| 0.030 | 25.1892 | 2.7776 |
| 0.035 | 27.2431 | 3.8154 |
| 0.040 | 29.8561 | 4.1112 |
| 0.045 | 31.1452 | 4.5772 |
| 0.050 | 32.0002 | 5.2681 |
| 0.055 | 33.5231 | 6.3314 |
| 0.060 | 34.1272 | 6.7821 |
| 0.065 | 35.4139 | 7.1957 |

Table 4: Variation in TC with the variation in K

| K | T | TAC(10^5) |
|--------|---------|---------------|
| 0.0008 | 32.8081 | 4.5417 |
| 0.0009 | 32.8081 | 4.9857 |
| 0.0010 | 32.8081 | 5.2681 |
| 0.0015 | 32.8081 | 6.8934 |
| 0.0020 | 32.8081 | 6.9572 |
| 0.0025 | 32.8081 | 7.2231 |
| 0.0030 | 32.8081 | 7.8573 |
| 0.0035 | 32.8081 | 9.4315 |
| 0.0040 | 32.8081 | 10.5473 |
| 0.0045 | 32.8081 | 11.1892 |

CONCLUSION

We developed a model with supplier’s trade offer of credit and price discount the purchase of stock. The model considered the both, deterioration effect and time discounting. Generally, supplier offer different price discount on purchase of items of retailer at different delay periods. Suppliers allow maximum delay period, after which they will not take a risk of getting back money from retailers or any other loss of profit. Constant deterioration is not a viable concept; hence, we have considered an inventory with deterioration increasing with time. The model presents ample scope for further extension and development. This study may be extended to multi-items. Another possible extension of this study may consider the assumption of the stochastic demand and deteriorate rate.

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